Schweizerische Eidgenossenschaft
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Swiss Confederation

Federal Department of Home Affairs FDHA
Federal Office of Meteorology and Climatology MeteoSwiss

Prerequisites - Models, Meshes, Vector Analysis

Overview:

- Intro model (FVM) and meshes.
- Prerequisites in vector analysis

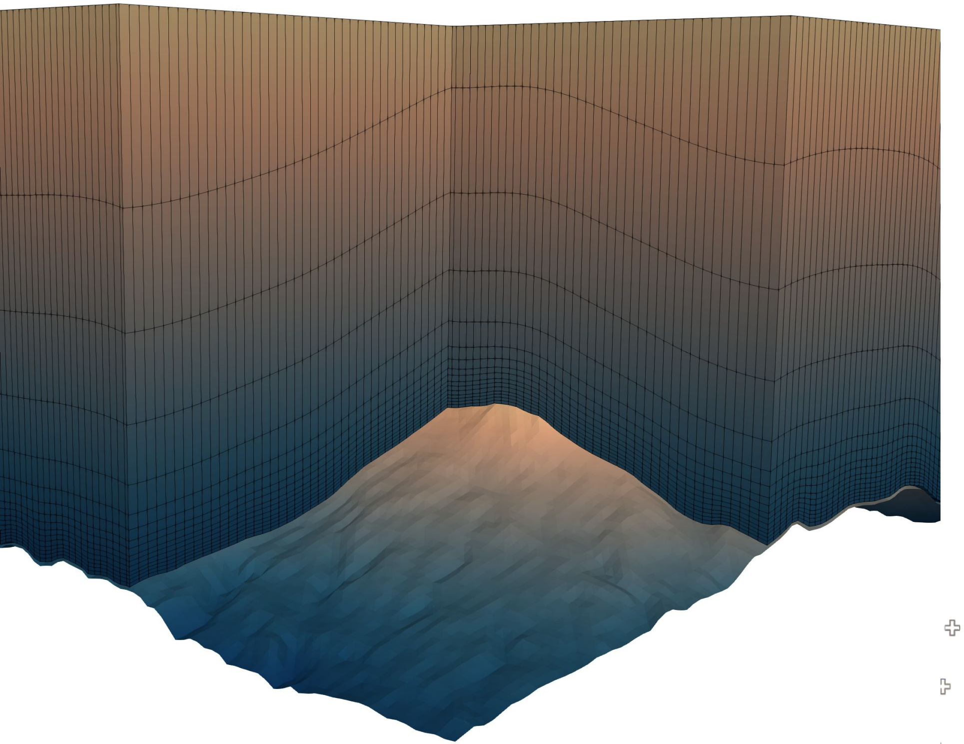
Intro: Models & Meshes

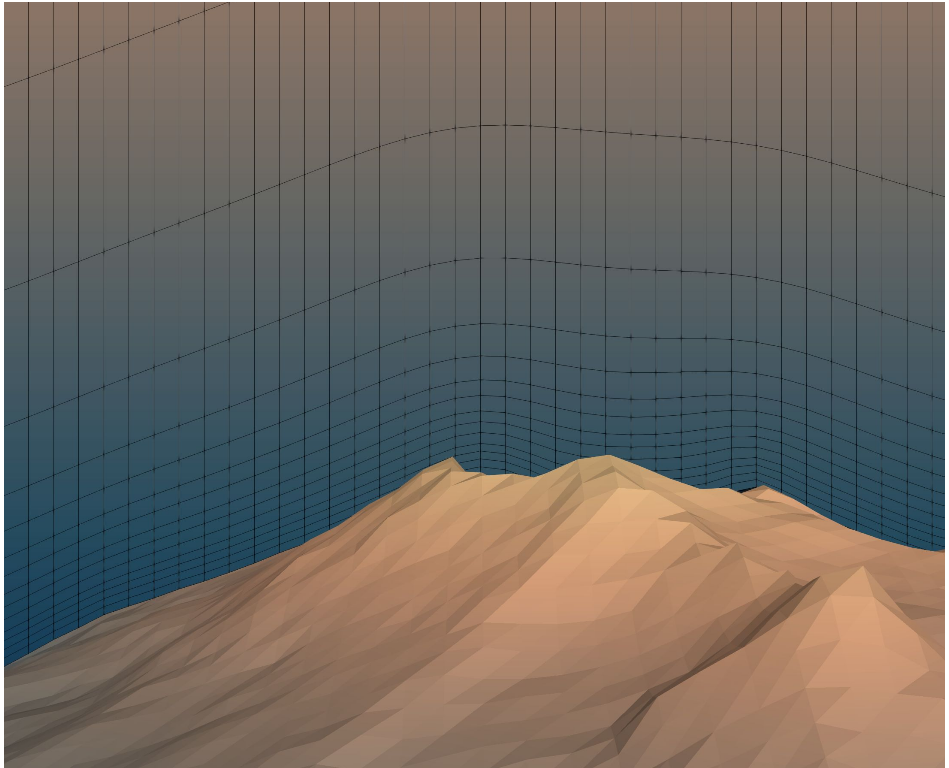
- Different NWP and climate models use different numerical paradigms. The most important ones being
  - Finite Differences
  - Finite Volumes
  - Spectral Methods
  - Discontinuous Galerkin Methods
- These numerical methods need different kinds of meshes.
  - Finite Differences → Cartesian Mesh
  - Finite Volumes → General (unstructured) Meshes (within reason)
  - ….

Intro: Models & Meshes

Meshes used in climate & NWP typically have some other interesting properties

- They are arranged in columns:





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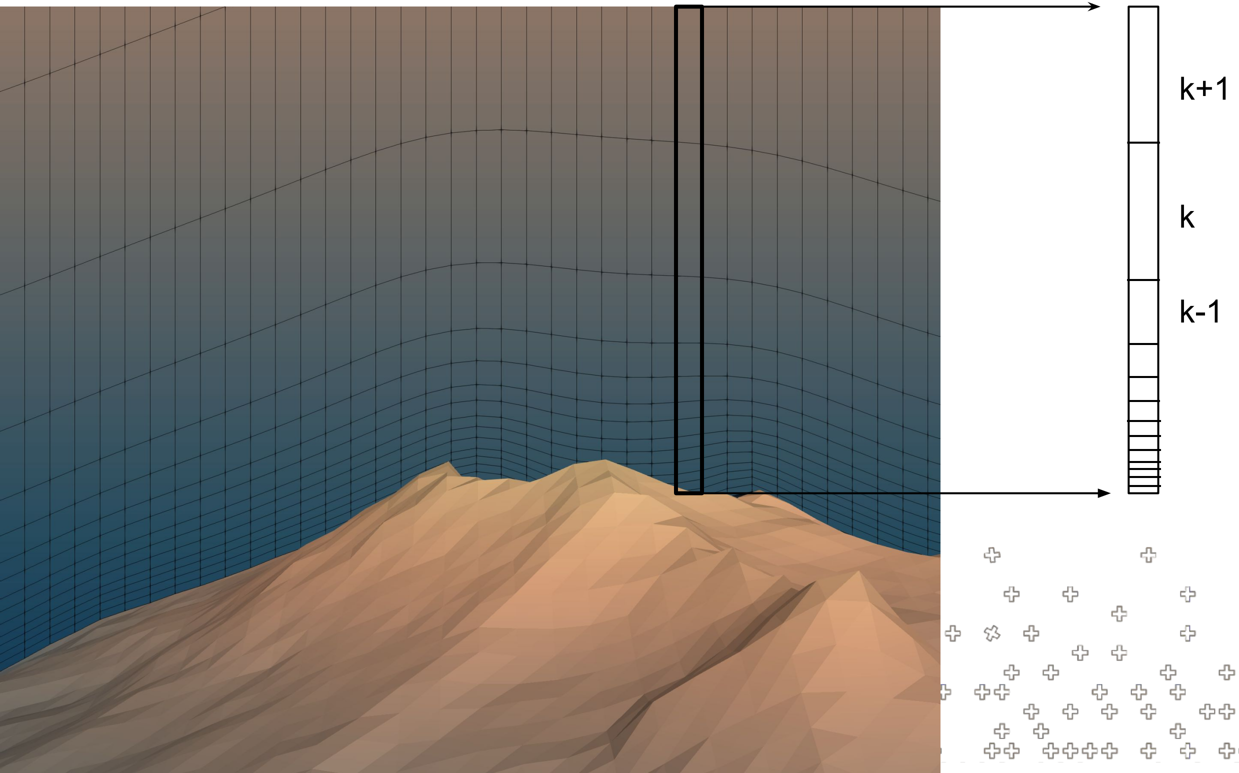
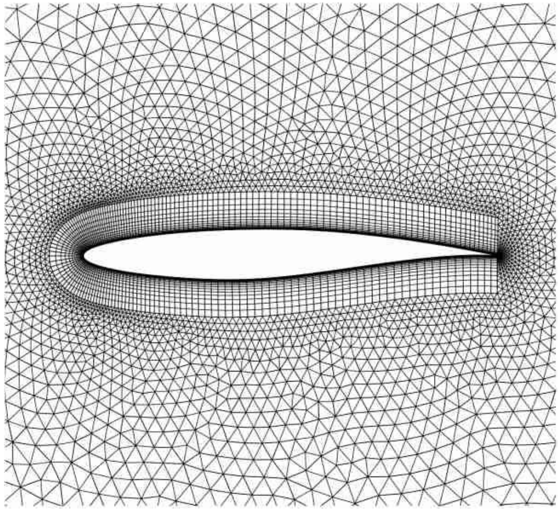


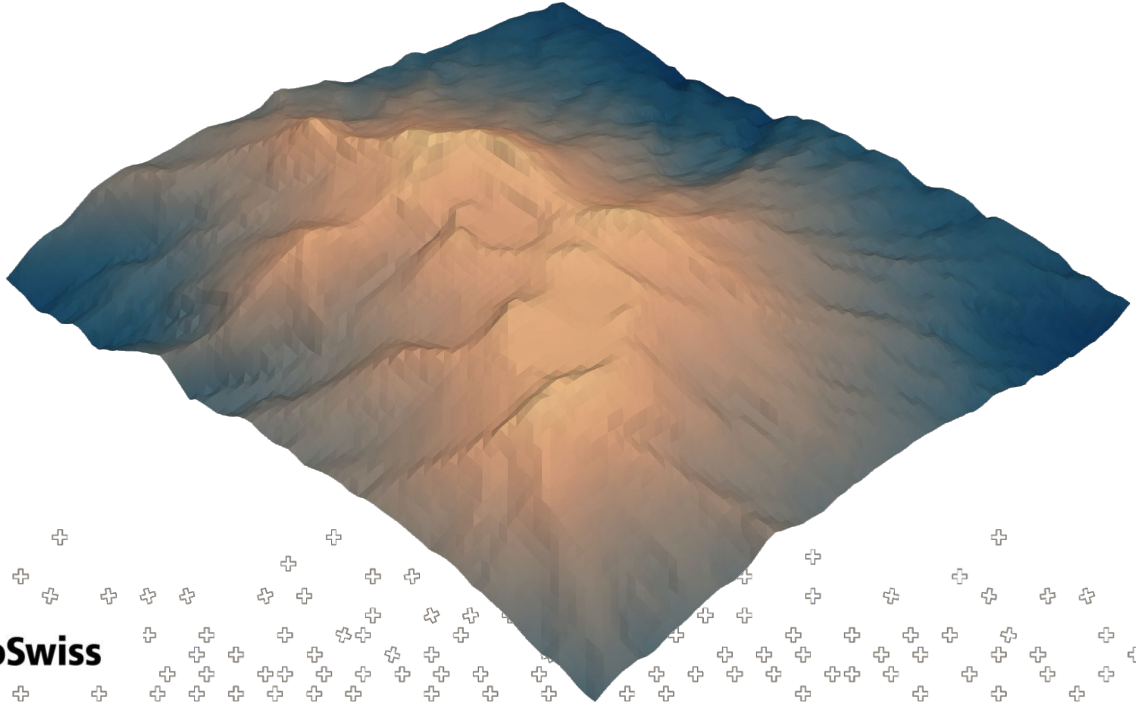


NOT true in general CFD

Intro: Models & Meshes

Meshes used in climate & NWP typically have some other interesting properties

- They follow the Terrain



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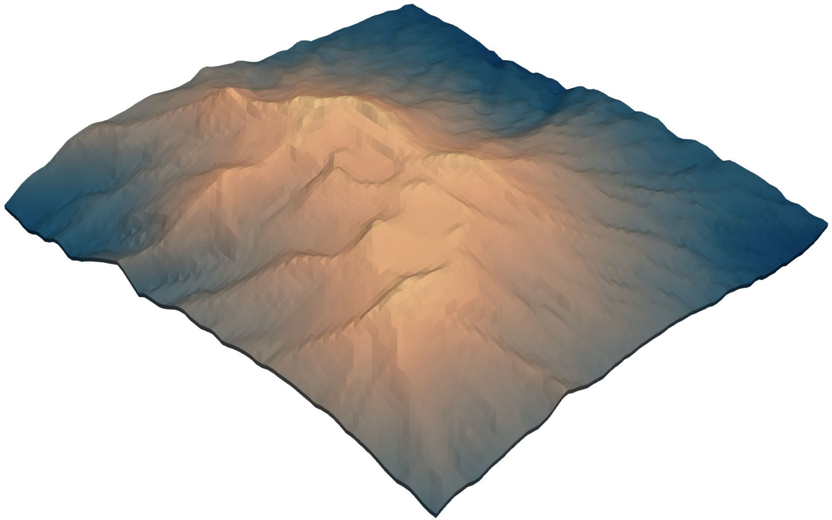

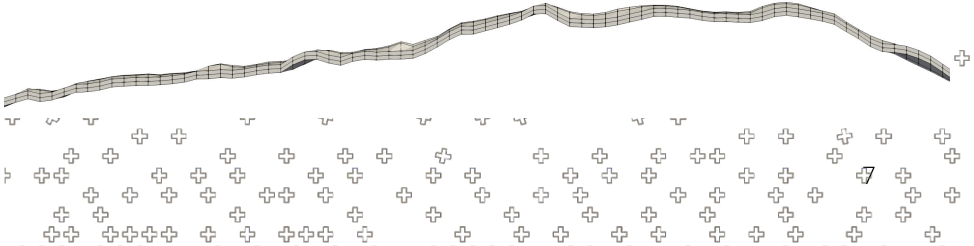

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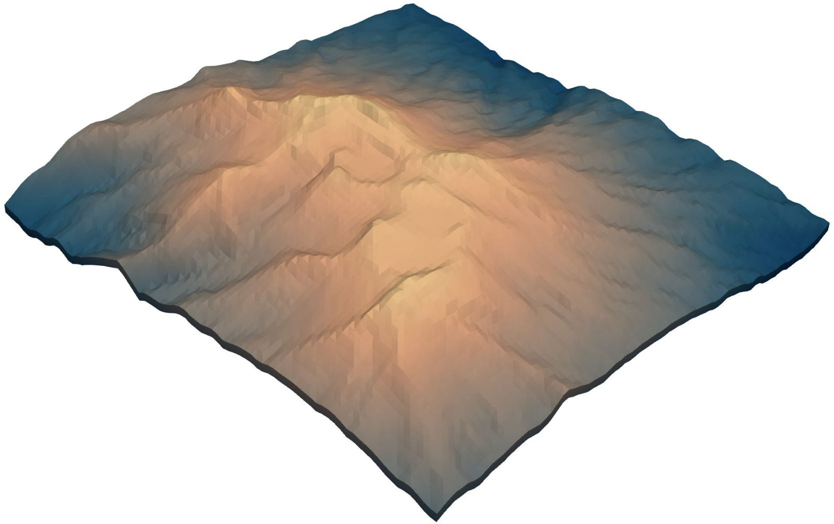

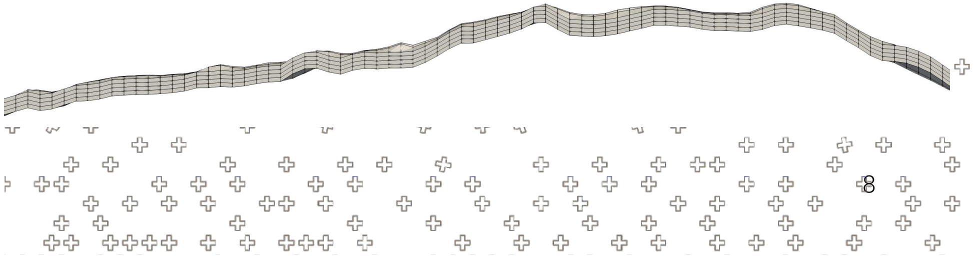

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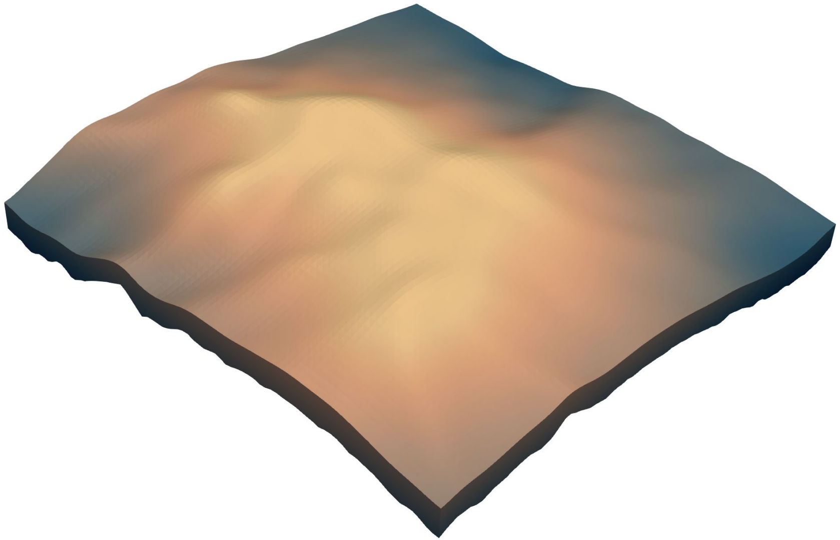

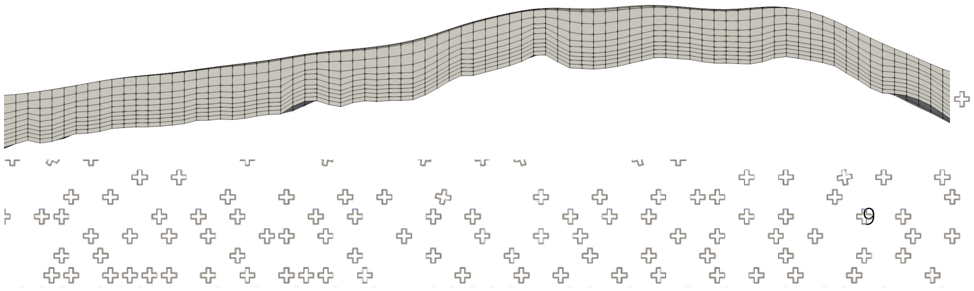

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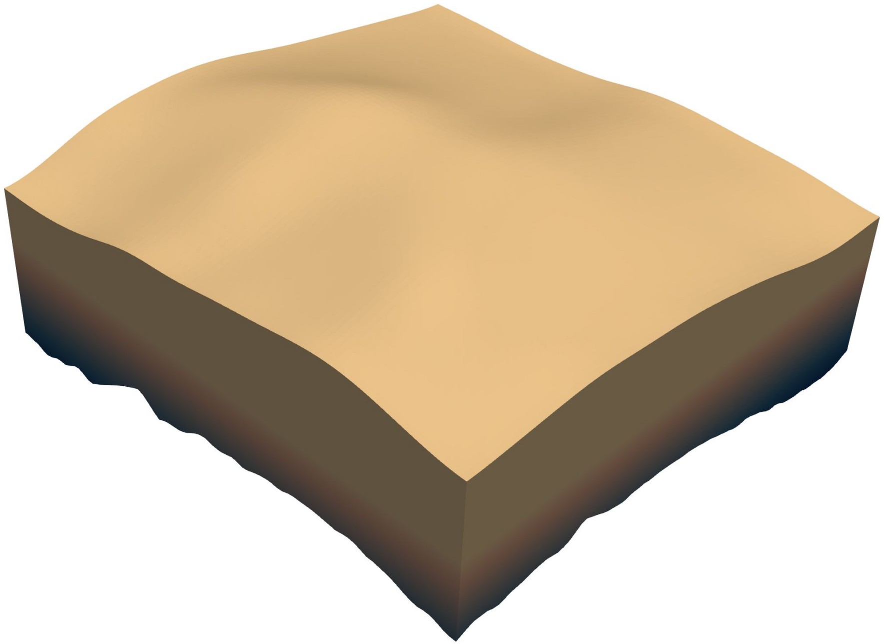

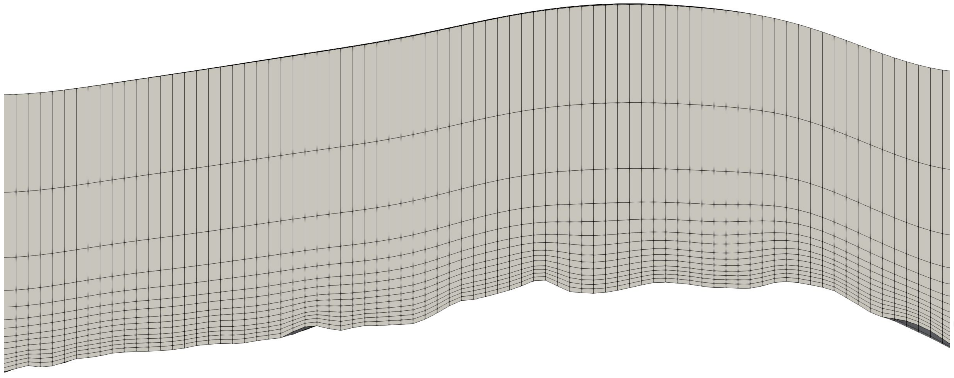

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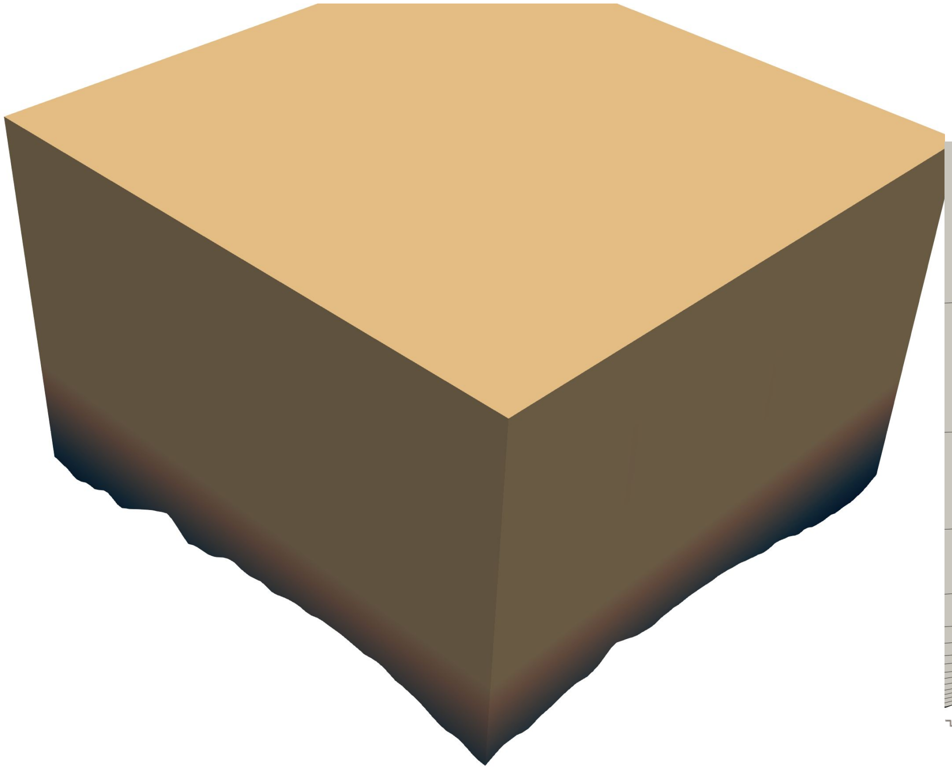
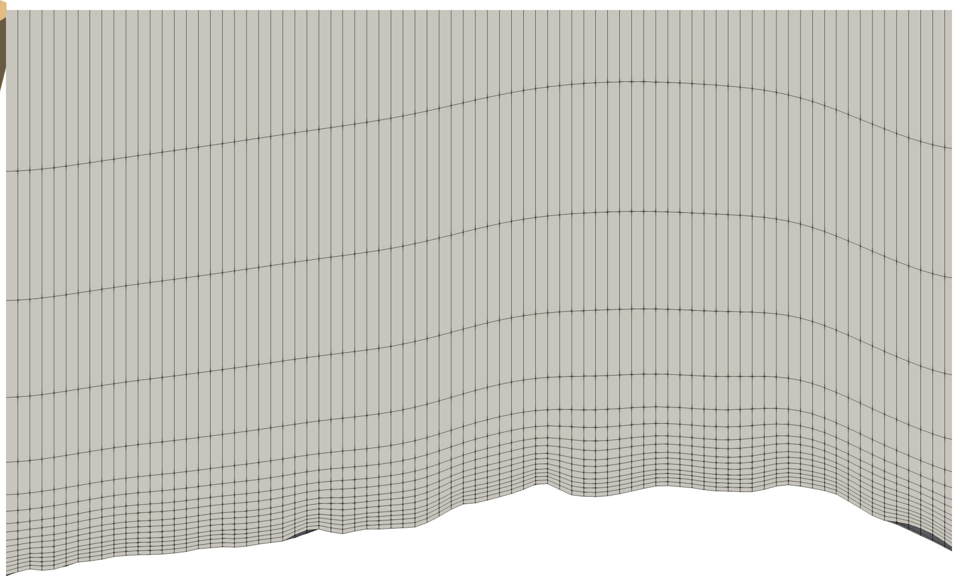
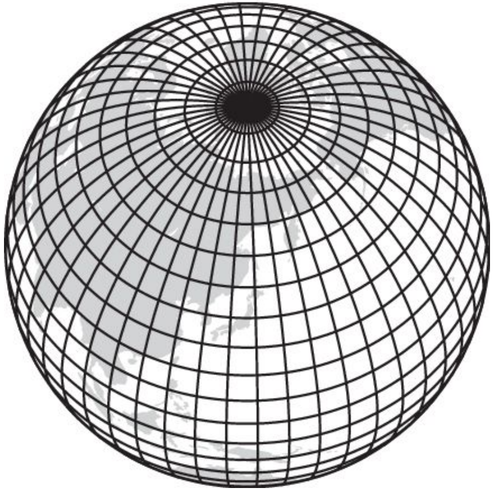




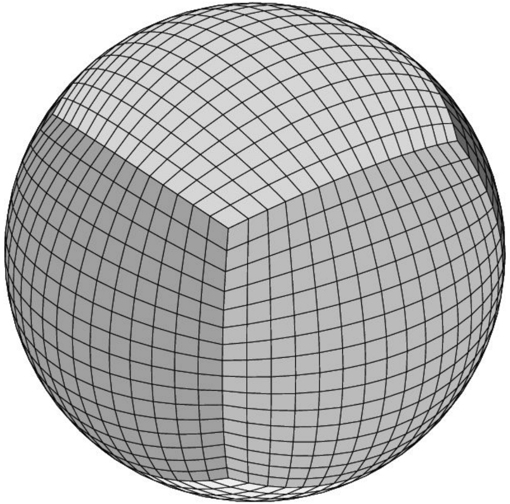
Intro: Models & Meshes

Lat/Lon Grid



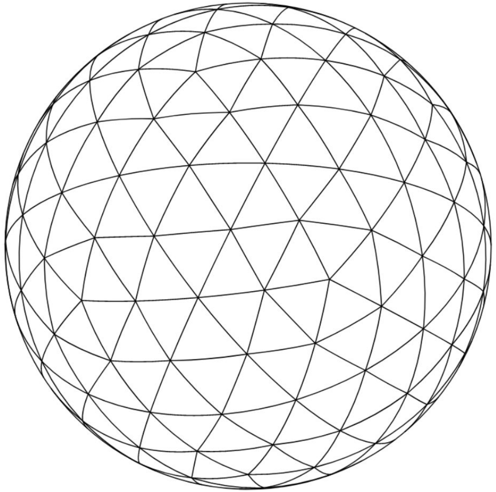
https://dx.doi.org/10.2151/jmsj.85B.241

Cubed Sphere



https://doi.org/10.5194/gmd-7-3017-2014

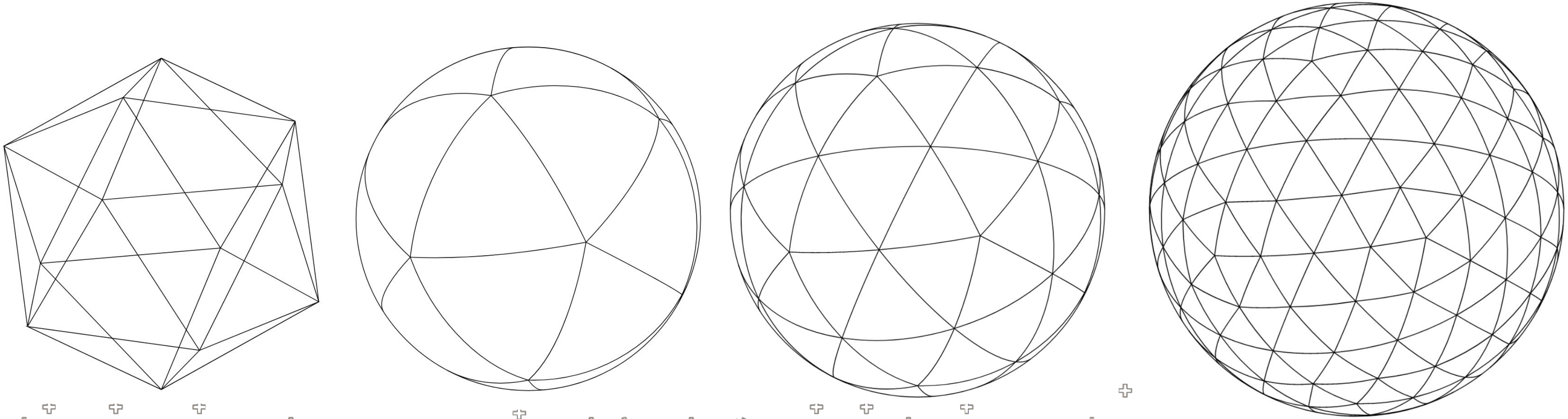
Icosahedral



https://doi.org/10.1186/s40668-020-00033-7

Intro: Models & Meshes - The ICON model

- ICON (Icosahedral Nonhydrostatic) Model
- Developed by DWD, MPI and others
- Model that drives dusk & dawn development
- Uses a special kind of triangular mesh → the Icosahedral mesh



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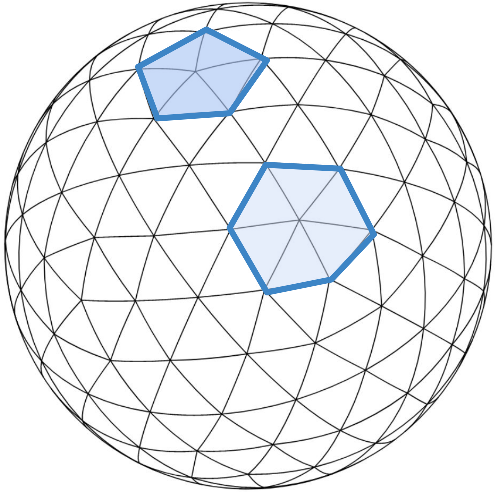
Intro: Models & Meshes - The ICON model

- ICON (Icosahedral Nonhydrostatic) Model
- Developed by DWD, MPI and others
- Model that drives dusk & dawn development
- Uses a special kind of triangular mesh → the Icosahedral mesh
- Very "close" to a structured hex-triangle mesh
  - dual grid is hexagonal everywhere
  - except on the corners of the original subdivision where the dual mesh is pentagonal



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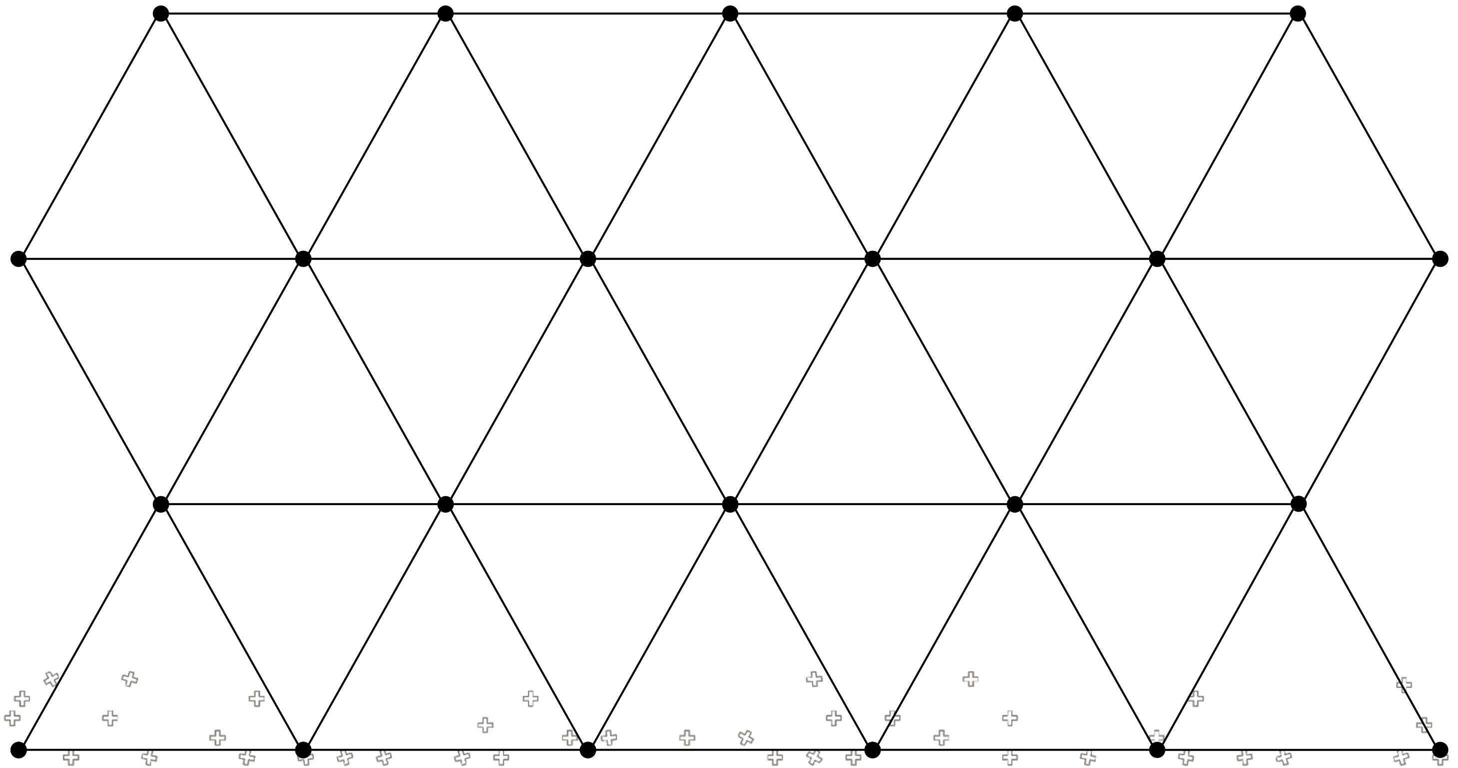
Intro: Models & Meshes - Hexa Triangular Meshes



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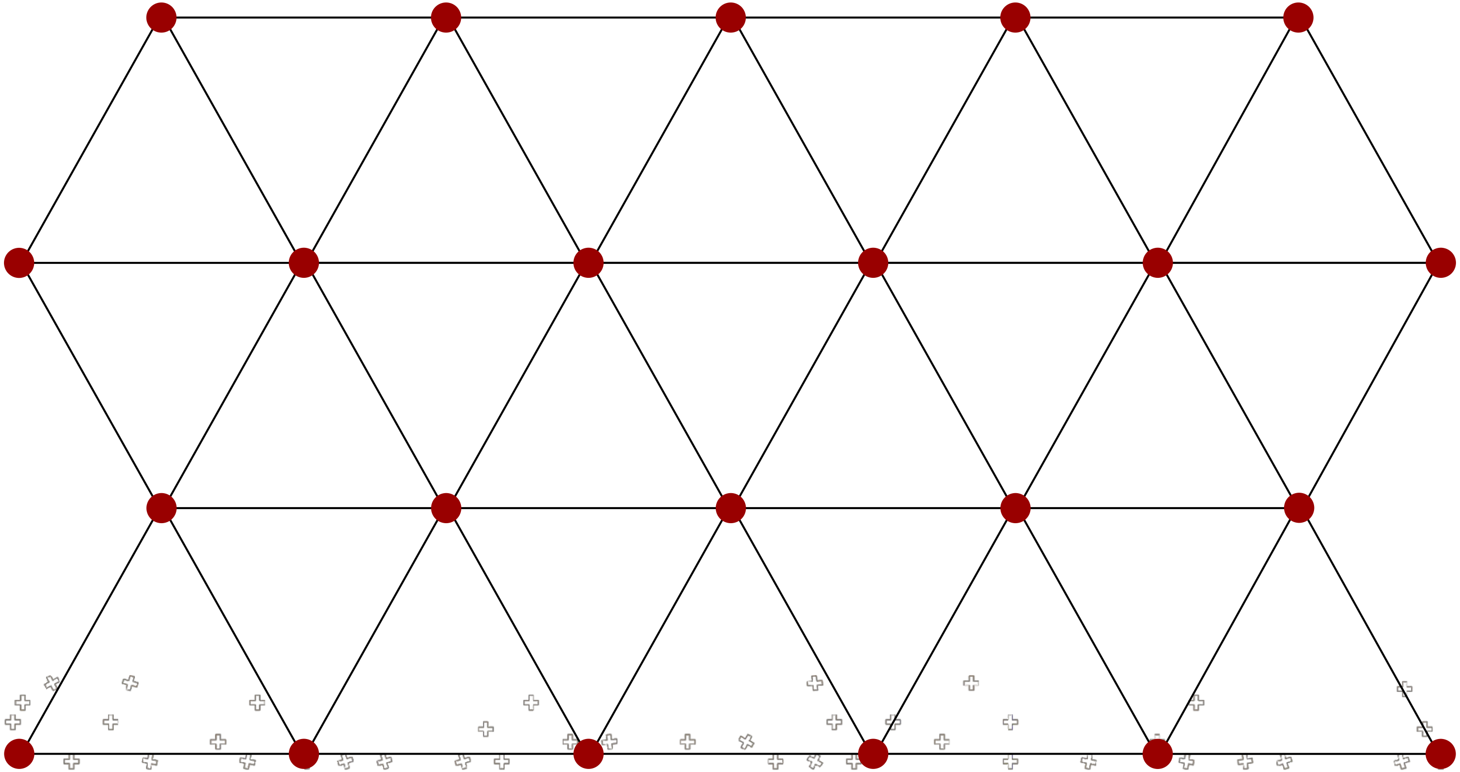

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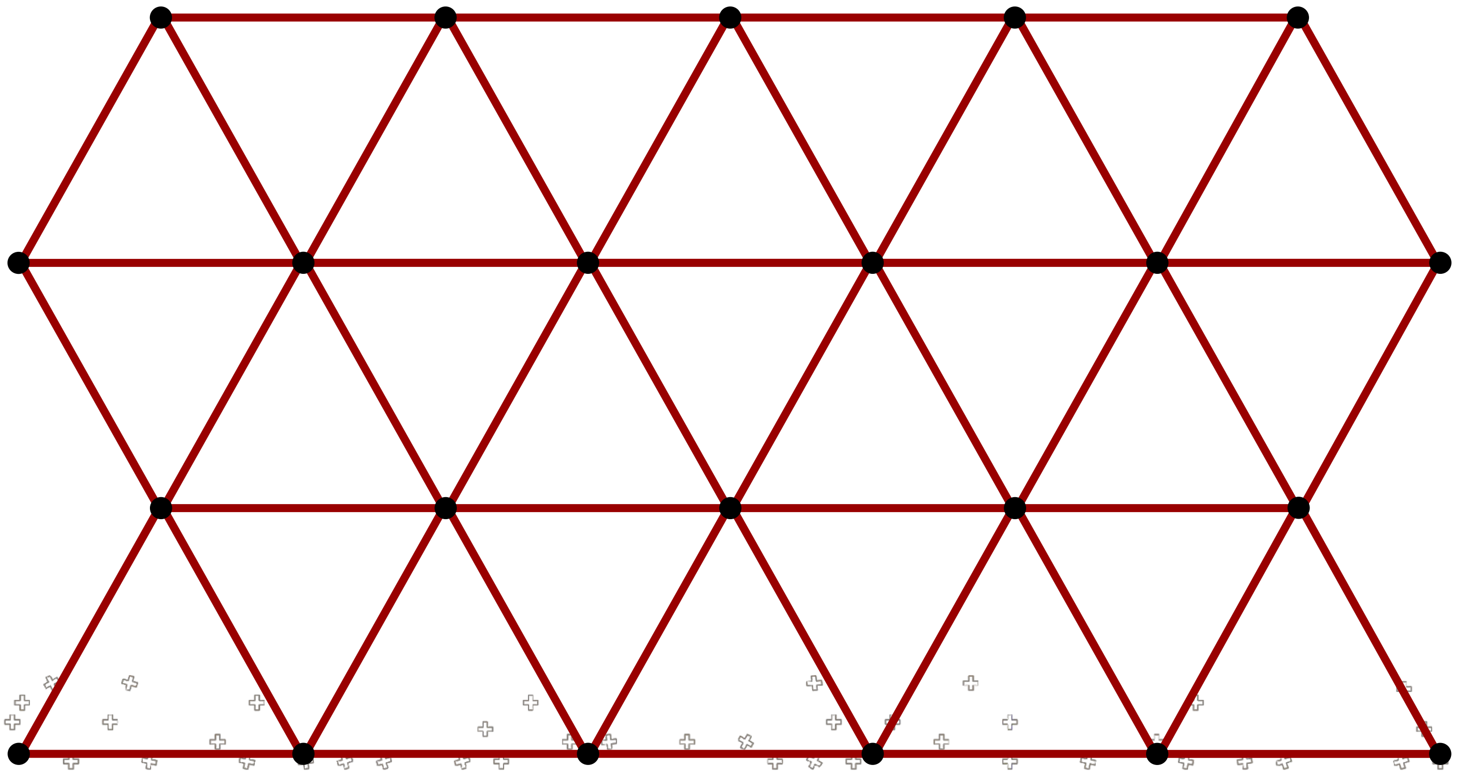

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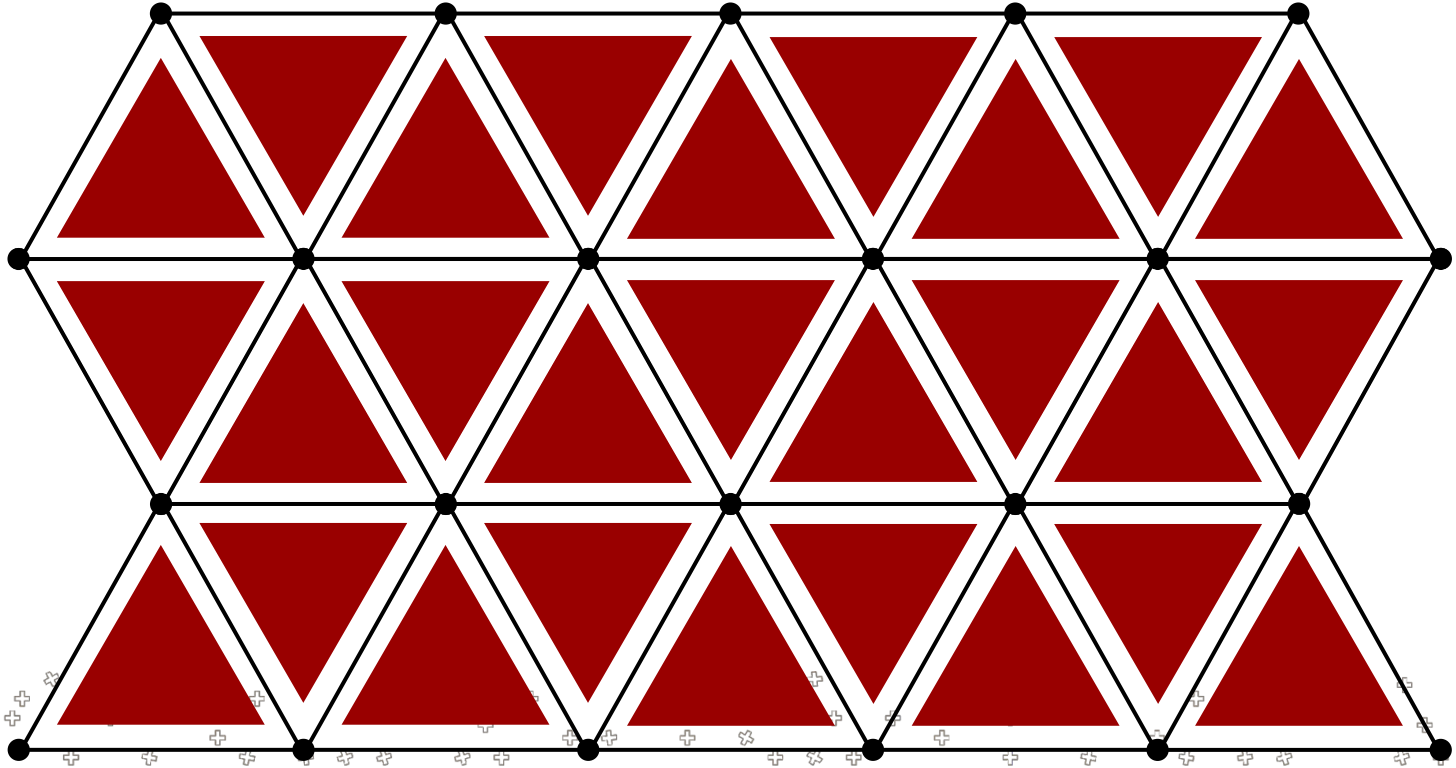
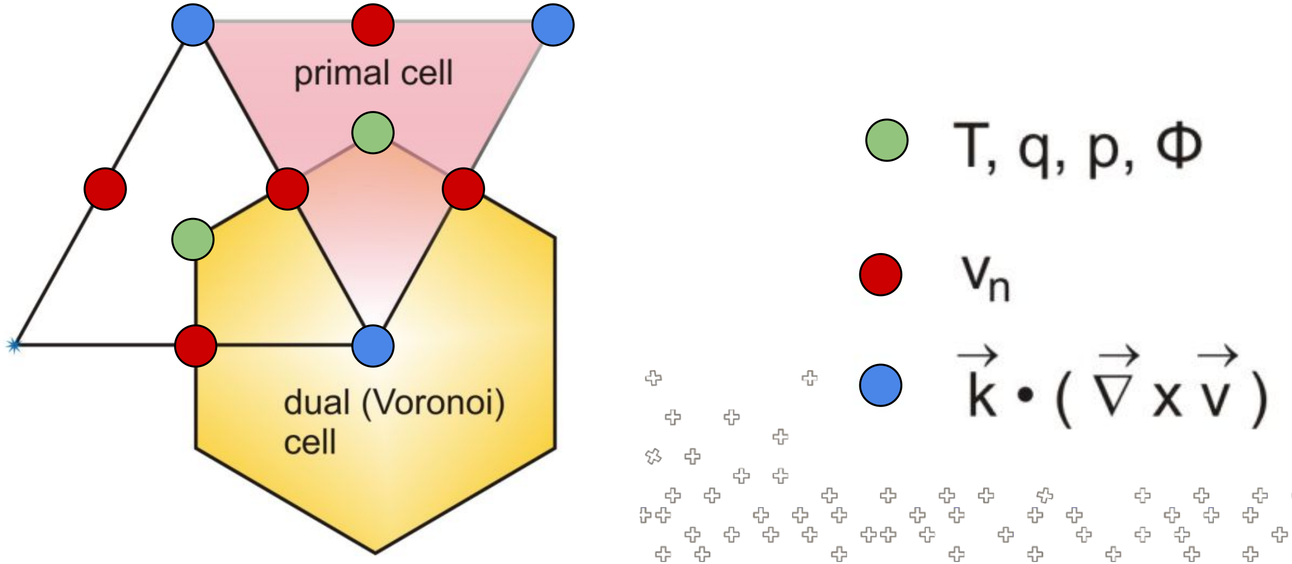


Intro: Models & Meshes

- Locations on mesh: Edges, Cell, Vertices (=Nodes)
- In Finite Volume (and to some extent Finite Difference) Models
  - Fields can be stored on any of the three locations
  - Where a field is stored is important
  - For example in ICON variables are located indicated below:



Vector Analysis Basics - The Gradient

In General:

$$\text{grad } f = \lim_{\Delta v \to 0} \frac{\oint_\Sigma f \, d\mathbf{S}}{\Delta v}$$

- Σ: closed surface
- Δv: enclosed volume

In Cartesian $\mathbb{R}^3$:

$$\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}(x,y,z), \frac{\partial f}{\partial y}(x,y,z), \frac{\partial f}{\partial z}(x,y,z) \right)$$

This can be intuitively understood as follows: dS is a boundary element (with outer unit normal) of the volume Δv with boundary Σ. The outer normal on Σ is scaled with the local value of the field f at every point and the scaled normals are integrated over the whole boundary of Δv. If f has the same value everywhere, the integral evaluates to zero. If f is larger on one side of Δv than on the other, the result will be a non-zero vector pointing from the side where f is smaller to the side where it is larger. If we let the size of the volume Δv go to zero, we obtain a vector indicating the local point change in f and its direction.

taken from lecture notes to Spatiotemporal Modeling and Simulation - Ivo Sbalzarini

Vector Analysis Basics - The Divergence

In General:

$$\text{div }\mathbf{v} = \lim_{\Delta v \to 0} \frac{\oint_{\Sigma} \mathbf{v} d\mathbf{S}}{\Delta v}$$

- Σ: closed surface
- Δv: enclosed volume

In Cartesian $\mathbb{R}^3$:

$$\nabla \cdot \mathbf{v}(x,y,z) = \left(\frac{\partial v_1}{\partial x}(x,y,z), \frac{\partial v_2}{\partial y}(x,y,z), \frac{\partial v_3}{\partial z}(x,y,z) \right)$$

This can be intuitively understood: the vector field is locally projected onto the outer unit normal of the boundary Σ of the volume Δv and all the projections are integrated over the whole closed boundary. The integral will thus evaluate to the total amount of **v** that is crossing the boundary. If we let the size of the volume Δv go to zero, we obtain the local amount (scalar) of **v** that is “emerging” out of a point in space.

Vector Analysis Basics - The Curl

In General:

$$\text{curl}\,\mathbf{v} = \lim_{\Delta v \to 0} \frac{\oint_{\Sigma} d\mathbf{S} \times \mathbf{v}}{\Delta v}$$

- Σ: closed surface
- Δv: enclosed volume

In Cartesian $\mathbb{R}^3$:

$$\nabla \times \mathbf{v}(x, y, z) = \left(\frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z}, \frac{\partial v_1}{\partial z} - \frac{\partial v_3}{\partial x}, \frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y} \right)$$

The intuitive meaning of this equation is the following: at every point on the boundary of the volume Δv, we compute the cross product between the local vector field value and the outer unit normal on the volume's boundary Σ. This cross product will be maximum if **v** is tangential to Σ and zero if **v** is parallel to d**S**. We then integrate this quantity over the whole boundary, thus measuring the net amount of **v** that is "running around" Σ. If we let the size of the volume go to zero, we obtain a vector whose length is the point-wise local vortex strength or rotation of the vector field **v** and whose direction indicates the axis of rotation.

taken from lecture notes to Spatiotemporal Modeling and Simulation - Ivo Sbalzarini

Vector Analysis Basics - The Curl

Operator	Symbol	Argument	Result	Interpretation
gradient	∇f	scalar	vector	steepest ascent
divergence	$\nabla \cdot \mathbf{v}$	vector	scalar	source density
curl	$\nabla \times \mathbf{v}$	vector	vector	vortex strength

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Vector Analysis Basics - Gauss Theorem

Also known as Green-Gauss Theorem or Divergence Theorem

$$\oint_{\delta B} \mathbf{v} \cdot \mathbf{n} \, dS = \int_{\mathbf{B}} \nabla \cdot \mathbf{v} \, dV$$

- The flux of **v** through ∂B from inside to outside is equal to the volume integral of $\nabla \cdot$ v over the enclosed volume B.
- “What is produced inside B has to flow out.” This is a consequence of the conservation of mass in a flow with velocity field **v**.

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Q&A

Questions?